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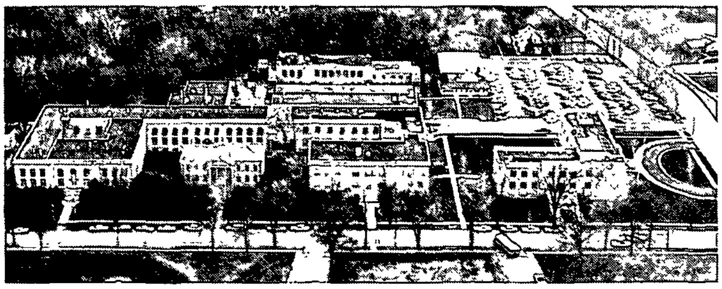

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Z-DIRECTION VARIATION OF INTERNAL STRESS AND PROPERTIES IN PAPER

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INTRODUCTION

There is a growing awareness of the importance of both in-plane and out-of-plane strength properties with respect to the converting and end-use requirements of paper. The in-plane strength properties of paper are relatively easy to measure mechanically, and recently, nondestructive ultrasonic wave propagation techniques have been developed to measure the elastic constants of paper (1). Out-of-plane properties are, by comparison, much more difficult to measure mechanically. There is still a need in this area for reliable and less time-consuming measurement techniques. Ultrasonic wave propagation techniques have also been developed to measure the out-of-plane elastic constants (2). One of the goals of nondestructive measurement techniques is to be able to predict both in-plane and out-of-plane failure and other strength related properties. A number of predictions have already been established, including tensile strength, compressive strength, and out-of-plane or z-direction tensile strength (3).

With the above goal in mind, Waterhouse (4) investigated the out-of-plane shear deformation behavior of paper employing a torsion mode. During the course of this investigation, where both mechanical and ultrasonic measurements were made, a significant variation in both in-plane and out-of-plane properties was found as material was symmetrically removed from linerboard and medium samples by surface grinding. The implication was that in-plane and out-of-plane elastic properties varied from plane to plane in the thickness direction. Furthermore, there was a difference (which has yet to be explained) in the variation of out-of-plane shear modulus in the thickness direction when measured mechanically and ultrasonically (4).

It is well known that paper is two sided, i.e., there is a felt and wire side. This arbitrary division is usually attributed to differences in fiber orientation and fines distribution within these regions. Less attention has been given as to how properties generally vary in the thickness direction and the factors controlling them. In addition to formation, wet pressing and drying conditions are possible factors which could influence property variations in the thickness direction. Further property modifications are possible during other operations, e.g., supercalendering. With regard to converting and end-use requirements, the most favorable distribution of properties in the thickness direction has yet to be determined.

One of the major difficulties which arises in determining the uniformity of composition or properties in the thickness direction is that of splitting or sectioning the sheet into layers. Parker and Mih (5) (who developed the Beloit sheet splitter), list various methods which have been used for this purpose, including razor blades, grinding, peeling with adhesive tape, and microtoming. In addition to the problem of obtaining uniform sections with minimal damage, there is concern that the properties of a particular section may be different from the properties of that same section in the whole sheet. The properties may change due to a release or change in internal stress. It is not expected that the sectioning process, and as a consequence changes in strength related properties, will have any effect on the composition of that section, i.e., filler, fines content, fiber orientation, or formation. Parker and Mih (5) used the Beloit sheet splitter to demonstrate among other things, ink penetration during printing, fines (filtration resistance) and filler (ash) distribution in the thickness direction. Fines distribution is clearly dependent on the type of forming method used. Parker and Mih (5) found that fines were more highly concentrated on the wire side of handsheets, whereas the maximum concentration of fines was on the felt side of fourdrinier produced paper. One disadvantage when using the Beloit sheet splitter is having to wet out the samples, since this precludes any meaningful measurements of strength related properties on the split sections. Mechanical properties on the redried sections would be different than in the original paper because of different drying conditions.

Kallmes (6) used the Beloit sheet splitter and IPC zero span tester to determine the z-direction variation of fiber orientation. He argued that the rewetting and drying of the samples should have a negligible effect on the MD/CD zero span ratio. Kallmes found, with a few exceptions, that the wire side of the sheet tends to be more square than the felt side of the sheet for both commercial sheets and those formed on an experimental former. In the experimental sheets it was also found that the greater the fiber orientation of the whole sheet, the greater the difference in orientation between the felt and wire sides of the sheet. Another interesting finding by Kallmes (6) on commercial sheets which had been split into four sections, was that fiber orientation was higher in the middle sections of the sheet. A similar effect was reported by Waterhouse (4), who found an increase in elastic modulus anisotropy in commercial linerboard and medium samples toward the middle of the sheet. It was suggested that the increase in anisotropy was due to variations in drying stress in the thickness direction, i.e., the interior layers of the sheet experienced a lower drying stress than the outer layers.

Wet pressing is an important process step in papermaking which serves not only to remove water from the web, but to consolidate it. Strength development through interfiber bonding is one of the main consequences of the consolidation process which is still not completely understood. Wickes (7) has demonstrated that interfiber bonding may not be uniform in the thickness direction. Using laminated wet webs and blotter stock as the wet press felt, he found a significant variation in both solids content and apparent density from layer to layer in the thickness direction. With two sided water removal, the distributions were still nonuniform but symmetrical. No results were reported for commercial wet press felts, and no other physical property measurements were made. It is also possible that nonuniformities in bonding from layer to layer in the thickness direction might be caused by internal damage due to too high a rate of water removal, in some instances giving rise to crush. Movement of fines and nonuniformities in the thickness direction during consolidation have been discussed by McGregor (8). He refers to this effect as sheet stratification, which he defines "as the change in vertical distribution of sheet fiber and filler material resulting from fluid shear force development during the dynamic wet pressing process". In summary we can say that wet pressing can influence the web structure in the following ways:

1) Uniformity of consolidation in the thickness direction
2) Stratification, i.e., movement of fiber and fines material due to fluid shear
3) Internal sheet disruption with excessive rates of water removal
4) Surface disruption dependent on felt type, press configuration and water removal rates

In the drying process water remaining mainly in the cell wall of the fibers is removed. This is accompanied by large changes in the dimensions of the fiber's cross section. These dimensional changes are communicated to other fibers in the network through interfiber bonding, and exert a considerable influence on the network's in-plane and out-of-plane deformation behavior, depending on the type of restraint applied during the drying process. In effect, the fiber's deformation behavior is modified, i.e., a sheet which is dried without restraint will consist of fibers which are microcompressed, the extent of which will depend on the level of interfiber bonding. Differences in the machine and cross machine direction restraint conditions on a paper machine are well recognized as being, in part, responsible for the anisotropy in various sheet properties.

If the sheet is restrained during the drying process, it is possible to monitor the drying stress it experiences. Htun (9) and others have shown that the deformation behavior of the network can be correlated with drying stress. Using the stress relaxation technique first used by Johansson and Kubat (10) to measure internal stress in paper, Htun also showed that drying stress is equal to internal stress. He also demonstrated that the level of drying stress is dependent on the viscoelastic nature of paper, i.e., the deformation behavior will be dependent on the drying conditions.

Returning to our main consideration of property variations in the z-direction, it is possible that even with a web having uniform composition, fiber orientation, and bonding, there can still be variations in drying and internal stress and related physical properties in the z-direction. Htun (9) discussed various definitions of stress. In keeping with the broader perspectives of materials science it is recommended that residual stress or internal stress are the more fundamental terms we should employ for paper. The term residual stress appears to be the one most commonly employed in the literature. The level and variation of internal stress in other materials, e.g., glass, wood, plastics, metals, adhesives, are of considerable concern to material scientists. Techniques for the measurement of internal or residual stress include photoelasticity, ultrasonics (stress-acoustic effect and surface Raleigh waves), stress relaxation, hole drilling, x-ray diffraction, eddy current, and layer removal employing strain gage or curvature measurements. Some of these methods are nondestructive, while others are only applicable to specific materials.

The main objectives of the present work are to measure the variation of internal or residual stress and associated properties in the z-direction. An attempt has been made by Lindroos and Waterhouse (11), to directly measure the variation of drying stress in the z-direction, and we hope to report on those results elsewhere.

RESULTS AND DISCUSSION

Z-Direction Variation of Internal Stress

The equivalence of internal and drying stresses, as discussed above, means in principle that it should be possible to measure the variation of drying stress in the z-direction by measuring the z-direction variation of internal stress, an idea proposed by Wiley (12).

To explore this possibility commercial 42-lb/1000 ft^2 linerboard was used. Conditioned samples were characterized, and in-plane and out-of-plane moduli were measured ultrasonically (2,13). The results for the sixteen sheets are given in Table I. Samples for stress relaxation measurements consisting of the whole board, wire, middle or felt sections were produced by surface grinding using a technique developed by Wink (14,15). These samples were similarly characterized after surface grinding.

Table I. Characterization of commercial 42-lb linerboard (avg. 16 sheets).

Sample	Basis Weight, g/m^2	IPC Cal, (mm)	Density, g/cm^3	E_x, GPa	E_y, GPa	G_{xy}, GPa	E_z, GPa	R, (E_x/E_y)
Avg.	207.5	0.287	0.723	10.47	4.26	2.51	0.0459	2.47
S.D.	1.699	0.0038	0.139	0.308	0.306	0.064	0.00156	0.212
%CV*	0.82	1.35	1.92	2.94	7.18	2.53	3.40	8.58

*Where %CV - coefficient of variation; E_x, E_y, and G_{xy} are the in-plane machine direction Young's modulus, the in-plane cross machine direction Young's modulus, and in-plane shear modulus, respectively. E_z is the z-direction longitudinal modulus.

Internal stress determinations in the machine and cross machine directions for the whole board, using the stress relaxation procedure employed by Johansson and Kubat (10), are shown in Fig. 1. These authors argue that the slope F of the linear portion of the stress-log time curve should be a linear function of the applied initial stress σ_0. Furthermore, the intercept on the stress axis should be equal to the internal stress σ_i. Values of internal stress determined by linear regression analysis of the data shown in Fig. 1 are 7.72 Nm/g and 1.56 Nm/g for the machine and cross machine directions, respectively. Internal stress measurements similarly determined and other properties for the whole sheet, felt, middle and wire sections are given in Table II.

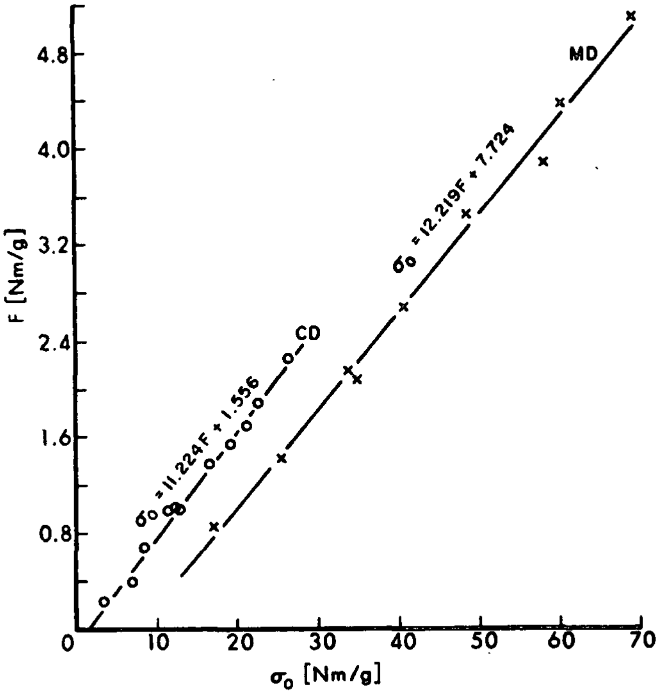

Fig. 1. Internal stress predictions from stress relaxation measurements.

There is indeed a significant variation in internal stress in the z-direction as seen in Table II. In the machine direction, the middle section has the lowest value of internal stress, while in the cross machine direction the wire side has the lowest value. The apparent density variation in the z-direction is not large. The out-of-plane longitudinal modulus is significantly lower in the middle section of the sheet, and a similar trend is found with the out-of-plane machine direction shear modulus. This behavior is similar to that reported by Waterhouse (4). It should be emphasized that, since these results are for a commercial linerboard, the interpretation is not unambiguous because of uncertainties with regard to composition, fines and fiber orientation distribution in the z-direction. The average properties of the felt, middle, and wire sections are generally less than those of the whole sheet. The question as to whether this may be due to damage from the surface grinding will be considered shortly. The stress relaxation method is viewed as an indirect method of determining internal stress and is rather time consuming. It would be of benefit to have a more direct measurement of internal stress to study its variation in the thickness direction.

When the various sections of the sheet were produced by surface grinding, a pronounced curvature development was observed. These are shown in Fig. 2. This curvature is a manifestation of the out-of-balance internal stress distribution which is created when the board is sectioned and is also direct evidence for the existence of internal stress variation in the z-direction.

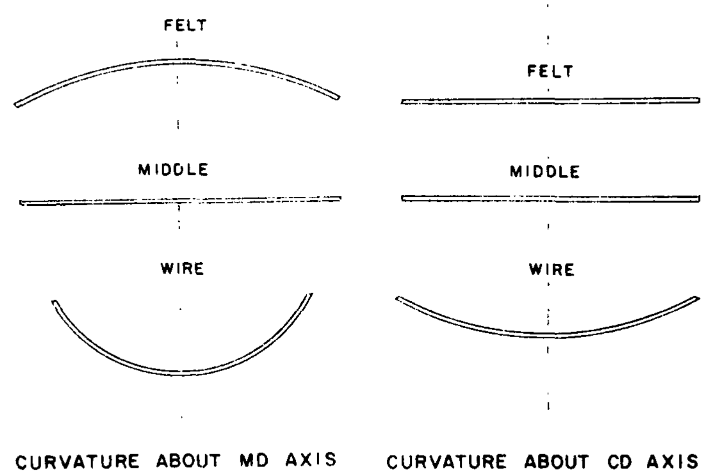

Fig. 2. Curvature measurements on 42-lb/1000 ft^2 linerboard sections.

Table II. Internal stress and other properties of 42-lb commercial linerboard.

Sample	Basis Weight, g/m^2	IPC Cal, mm	Density, g/cm^3	E_{MD}/ρ (km/sec)^2	E_{CD}/ρ (km/sec)^2	R	E_z/ρ (km/sec)^2	G_{xz}/ρ (km/sec)^2	G_{yz}/ρ (km/sec)^2	$\sigma_{i_{MD}}$ Nm/g	$\sigma_{i_{CD}}$ Nm/g
Felt side	94.1	0.1219	0.772	12.4	5.19	2.39	0.069	0.069	0.066	6.65	2.06
Middle	98.7	0.1358	0.727	11.4	5.04	2.27	0.043	0.068	0.066	3.95	3.12
Wire side	86.9	0.1191	0.729	12.1	3.81	3.17	0.059	0.077	0.068	5.61	1.10
Whole sheet	207.5	0.2870	0.723	13.1	6.23	2.10	0.064	0.106	0.086	7.72	1.56

Where G_{xz}/ρ and G_{yz}/ρ are the out-of-plane specific shear moduli and $\sigma_{i_{MD}}$ and $\sigma_{i_{CD}}$ are the specific machine and cross machine direction internal stresses.

A method for calculating the internal stress distribution, using curvature measurements has been developed. It is similar to that of Rybicki et al. (16), who used sectioning methods and strain measurements to calculate residual stress distributions in pipes and plates. The details are presented in Appendix A.

Radius of curvature, ultrasonic moduli, and calipers for the commercial linerboard sample are given in Table III.

Table III. Measurements for internal stress calculations.

Sample	Apparent Density, g/cm	Radius of Curvature, R cm	t mm	E/ρ $(km/sec)^2$
Felt side, MD	0.772	67.1	0.122	12.4
Felt side CD	0.772	8.08	0.122	5.19
Middle, MD	0.727	142.2	0.136	11.4
Middle, CD	0.727	28.2	0.136	5.04
Wire side, MD	0.729	8.46	0.120	12.1
Wire side, CD	0.729	3.68	0.120	3.81

Using this data and the equations given in Appendix A, we calculated the internal stress distributions shown in Fig. 3.

It is interesting to note that the center layer in both the MD and CD samples of the sheet is in tension, while the outside layers are in compression. Thermally toughened glass and polymers exhibit a similar behavior where the internal stress distribution is approximately parabolic, with the outside layers in compression and the center in tension.

Curvature and stress relaxation methods demonstrate a variation of internal stress in the z-direction. A significant variation in in-plane and out-of-plane properties has also been found. The equivalence of these two methods for internal stress distribution determinations and their relationship to drying stress has yet to be determined. In a recent review, Isayev and Crouthamel (17) question the meaning of internal stress measurements made using the stress relaxation technique, and they believe the method is unsuitable for determining the distribution of internal stress. The layer removal technique, together with curvature measurements, is preferred by Isayev and Crouthamel (17) for determining the internal stress distribution in polymer slabs produced by injection molding.

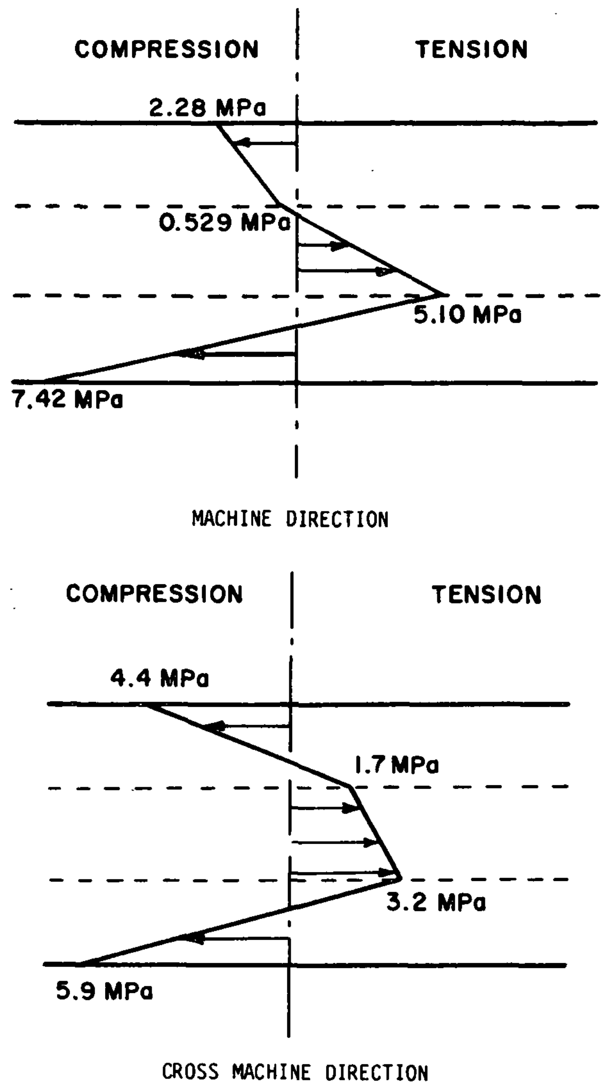

Fig. 3. Internal stress distrubtion for commercial 42-lb/1000 ft linerboard.

Let us give further consideration to Htun's (9) findings that the internal stress determined by stress relaxation is equivalent to the drying stress. When the drying process is completed, there is no resultant stress acting on the sheet (i.e., the drying stress has been reduced to zero), and therefore the average internal compressive stress, suitably computed, must be equal to the average internal tensile stress. Thus, it can be argued that the internal stress measured by Htun (9) should be equal to twice the average internal compressive (or tensile) stress measured using the curvature method.

A comparison of internal stress measurements made using stress relaxation (Table II) and curvature measurements (calculated from the stress distributions shown in Fig. 3) are given in Table IV.

Table IV. Comparison of internal stress measurements.

Direction	Stress Relaxation Method, Nm/g	Sectioning/ Curvature Method, Nm/g
MD	7.72	3.88 (2.29)
CD	1.56	3.09 (1.82)

The agreement, at least in magnitude, of the results by the two methods is encouraging, particularly in view of the assumptions made (see Appendix A) and the uncertainty in the interpretation of internal stresses as determined by the relaxation method. The calculations given in parentheses in Table IV are based on estimated values of elastic modulus for each of the sections at normal TAPPI testing conditions. In Instron tensile testing of the 42 lb/1000 ft^2 linerboard it had been found that $E_{Instron}/E_{Ultrasonic}$ = 0.59. It is possible that the layer removal/curvature technique and analysis (which can account for the biaxial nature of the internal stress system) used by Isayev and Crouthamel (15) might yield more accurate results.

The internal stress variation determined using the stress relaxation method (Table II) implies, according to Htun, that the drying stress is higher in the surface layers than in the interior of the sheet. Modulus measurements tend to support this contention. There appears to be a contradiction, however, inasmuch as the internal stress distributions determined from layer removal and curvature measurements suggest the opposite effect; that is, the outside layers, which are ultimately in compression, should be subjected to a lower drying stress than the middle layer.

One possible resolution of this paradox is suggested. During the early stages of drying, the surface layers may experience a greater tensile drying stress than the interior. The phenomenon of stress reversal is common in glass and other polymer systems. During this phase, relaxation processes and stress activated molecular reorientation can occur more readily (18,19), thus yielding greater increases in moduli in the surface layers of the sheet, even though when dry these layers are in compression.

The variation of properties in the z-direction has also been measured on handsheets having a random fiber orientation and a high degree of uniformity. One set of handsheets was made on the Formette Dynamique and dried at 91°C for 30 min. Another set was made on an IPC handsheet former and then press-dried with upper platen temperatures of 121°C, 177°C, and 232°C, respectively, and a lower platen temperature of 96°C. The press loading was 400 psi (2.76 MPa). Three handsheets were press-dried at each temperature level and, after characterization, were surface ground to produce top, middle, and bottom sections. Nondestructive characterization of the whole sheet and sections is given in Table V.

Table V. Characterization of Formette and press-dried handsheets. Unbleached southern pine, 600 CSF.

Sample	IPC Caliper, mm	Basis Weight, g/m^2	Apparent Density, g/cm^3	Specific Modulus E/ρ, km/sec	Specific Modulus E_z/ρ, km/sec
Formette					
F-1 Top	0.0789	66.2	0.839	8.49	0.211
F-3 Middle	0.1041	87.4	0.840	9.05	0.158
F-2 Bottom	0.0899	73.9	0.821	10.5	0.218
Whole Sheet	0.2922	230.9	0.790	9.65	0.334
Press Dried					
121°C					
Top	0.0948	104.3	1.100	8.97	0.284
Middle	0.1029	103.7	1.008	6.61	0.123
Bottom	0.0878	78.4	0.893	5.03	0.098
Whole	0.2138	214.9	1.022	8.71	0.418
177°C					
Top	0.0972	107.1	1.101	9.54	0.281
Middle	0.1045	108.1	1.034	6.46	0.124
Bottom	0.0887	77.1	0.870	5.28	0.0919
Whole	0.2162	214.4	0.992	9.94	0.413
232°C					
Top	0.0983	106.6	1.084	9.09	0.253
Middle	0.1068	108.6	1.017	7.3	0.138
Bottom	0.0896	77.2	0.862	5.24	0.0829
Whole	0.2164	212.8	0.967	10.2	0.410

The results show significant property changes in the thickness direction, particularly for the press-dried handsheets. The higher density of the Formette handsheet sections when compared with the whole sheet is due to a reduction in roughness with surface grinding; however, the differences in density of the sections is not significant. The bottom or low temperature sections of the press-dried handsheets have a significantly lower density than the top and middle sections. This is in part responsible for the lower values of in-plane and out-of-plane moduli, the latter showing the greatest change. It is not known whether the overall reduction in out-of-plane modulus is due to the effects of surface grinding or to a change in internal stress distribution. It is clear, however, that significant gradients in properties can be induced by the large temperature gradients experienced by the press-dried handsheets. The effects of possible damage induced by surface grinding will now be briefly examined.

Samples of 42-lb/1000 ft^2 commercial linerboard were subjected to three levels of grinding, with 0.089, 0.044 and 0.013 mm of material being removed per pass to produce felt, middle and wire sections. Properties of these sections and the whole sheet are given in Table VI. If the properties are graphed as a function of the amount of material removed by grinding, no clear-cut trends are indicated. Some properties increase and some decrease as the severity of cutting is reduced. Nevertheless, an attempt was made to extrapolate the various properties to zero mm removed/pass, and the results are summarized in Table VII. The extrapolated values again indicate that there is a significant variation of elastic properties in the z-direction, particularly the out-of-plane properties. The variation in radius of curvature is also indicative of a significant variation in internal stress.

Isayev and Crouthamel (17) also investigated the effects of machining and found significant differences with the type of milling machine employed. They were also able to demonstrate, using annealed samples, that no significant curvature was induced with the method of choice.

Table VI. Effects of surface grinding treatment on section properties.

	mm removed/ pass	Apparent Density, g/cm^3	V_{MD}^2 (km/sec)^2	V_z^2 (km/sec)^2	Caliper, mm	Radius of Curvature, cm
Felt	0.089	0.719	12.98	0.0499	0.1051	4.51
	0.044	0.716	12.64	0.0511	0.1184	5.31
	0.013	0.728	12.39	0.0545	0.1118	5.40
Middle	0.089	0.705	11.48	0.0328	0.1150	14.0
	0.044	0.688	11.63	0.0311	0.1029	19.8
	0.013	0.701	11.75	0.0340	0.1187	8.6
Wire	0.089	0.656	11.10	0.0452	0.1053	3.09
	0.044	0.665	11.89	0.0434	0.1102	2.95
	0.013	0.657	12.08	0.0503	0.1045	2.82
Whole sheet	--	0.689	12.64	0.0571	--	--

Table VII. Extrapolated properties of surface ground sections.

	Apparent* Density g/cm^3	V_{MD}^2 (km/sec)^2	V_z^2 (km/sec)^2	R_{cm}
Felt	0.721	12.3	0.0555	5.45
Middle	0.698	11.8	0.0350	14.1
Wire	0.659	12.2	0.0530	2.80
Whole Sheet	0.689	12.6	0.0571	--

*Average values.

CONCLUSIONS

The variation of properties in the thickness or z-direction of paper and their relationship to papermaking process variables have been reviewed. Special attention has been given to the drying process and its effect on drying and internal stresses. It is argued that a paper, having a high degree of uniformity in the thickness direction, i.e., fiber orientation and bonding, can still have, due to its viscoelastic nature, a significant drying and internal stress variation in the z-direction.

Two methods have been used to measure the variation of internal stress, and both involve measurements on felt side, middle, and wire side sections of paper produced by surface grinding. The first method involves stress relaxation measurements, while the second involves curvature measurements. A significant variation of internal stress and physical properties in the z-direction has been measured on samples of commercial linerboard, Formette handsheets dried at 91°C, and press dried handsheets dried at temperatures of 121°C, 177°C, and 232°C.

In an attempt to compare the two methods, we found an order of magnitude agreement for the internal stress on the commercial linerboard sample.

When we examined the severity of surface grinding on section properties, no common trend was found; however, it is recommended that, although time consuming, the minimum amount of material removed/pass should not exceed 0.013 mm.

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REFERENCES

1. Baum, G. A. "The Elastic Properties of Paper". Progress in Paper Physics Seminar. STFI Stockholm, Sweden, June 25-29 1984, to be published.
2. Habeger, C. "Measuring Z-direction elastic properties in paper with ultrasonic techniques". Progress in Paper Physics Seminar. STFI Stockholm, Sweden, June 25-29 1984, to be published.
3. Baum, G. A. Paper 19:65-67(1984).
4. Waterhouse, J. F. Tappi 67(6):104-108(1984).
5. Parker, J., Mih, W. C. Tappi 47(5):254(1964).
6. Kallmes, O. J. Tappi 52(3):482(1969).
7. Wickes, L. Tappi 65(9):124(1984)
8. MacGregor, M. A. Tappi 66(6):53(1983).
9. Htun, M. "The Influence of drying time and temperature on drying stress and related mechanical properties of paper". Doctoral Dissertation. Stockholm, Sweden, The Royal Institute of Technology, 1981.
10. Johansson, F., Kubat, J. Svensk Papperstid. 20:822 (Oct. 31, 1964).
11. Lindroos, S., Waterhouse J. F. unpublished work.
12. Wiley, J. H. "An investigation of Drying Stress Variation through the thickness of Paper". Student A-200 problem 399A, IPC June 16, 1982.
13. Baum, G. A. IPC Technical Paper Series Number 119, Dec. 1981.
14. Wink, W. A., Private Communication.
15. Beckman, N. J. and Plucker, E. I. TAPPI Coating Conference Preprint April 30-May 2, 1973.
16. Rybicki, E. F., Shadley J. R., Shealy W. S. Experimental Mechanics 438, Dec. 1983.
17. Isayev, A. I., Crouthamel, D. L. Polymer-Plast Technol. Eng. 22(2):177-232(1984).
18. Htun, M. Tappi J. 67(9):124(Sept. 1984).
19. Htun, M. "The Influence of Drying Strategies on Paper Strength" Progress in Paper Physics Seminar, STFI Stockholm, Sweden, June 25-29, 1984.

APPENDIX A

A procedure for calculating the internal stress distribution of paper using curvature measurements is given below. It is similar to that of Rybicki et al. (16) who used sectioning and strain gage measurements to calculate residual stress distributions in pipes and plates.

The analysis is developed for a board in three sections but is not limited to this number. The following assumptions are made.

1. the stress distribution is linear in each section
2. only elastic behavior is considered
3. the analysis is one dimensional and the stress distributions in the machine and cross machine directions are treated independently of each other.

Notation for the three sections is shown in Fig. A-1. The stress σ on each section is resolved into a tensile and bending stress. After sectioning, the tensile stresses go to zero and the bending contribution results in curvature of the section. The equilibrium force F and moment M equations for the three sections 1, 2, and 3 are:

$$F_1 + F_2 + F_3 = 0. \quad (1)$$

$$M_1 + M_2 + M_3 + F_1(t_1/2 + t_2 + t_3) + F_2(t_2/2 + t_3) + F_3\, t_3/2 = 0 \quad (2)$$

For each section the resultant stress is given by the following pair of equations.

$$\sigma_{1a} = F_1/t_1 + \bar{\sigma}_{1a}$$
$$\sigma_{1b} = F_1/t_1 - \bar{\sigma}_{1b} \quad (3)$$

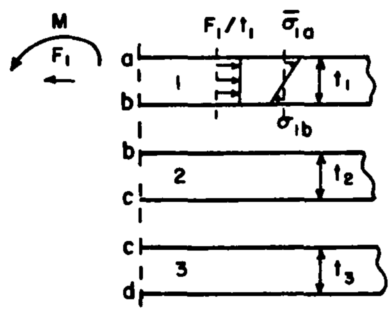

Fig. 1-A. Notation for internal stress analysis.

We also have the condition that

$$\bar{\sigma}_{1a} = \bar{\sigma}_{1b} = \bar{\sigma}_1 \qquad (4)$$

and similarly for sections 2 and 3.

Stress continuity between sections also requires that

$$\sigma_{1b} = \sigma_{2b}$$
$$\sigma_{2c} = \sigma_{3c} \qquad (5)$$

Using these conditions and equations (6) for the bending moment on each section, a set of equations for the determination of the unknown forces F_1, F_2, and F_3 can be derived. Knowing F_1, F_2 and F_3 the internal stresses can be calculated from equations (3) above.

$$M = \bar{\sigma}\, t^2/6 \qquad (6)$$

$$F_1A + F_2B + F_3C + \sum_1^3 \bar{\sigma}\, t^2/6 = 0$$
$$F_1 + F_2 + F_3 = 0$$
$$F_1/t_1 - F_2/t_2 - (\bar{\sigma}_1 + \bar{\sigma}_2) = 0$$
$$F_2/t_2 - F_3/t_3 - (\bar{\sigma}_3 + \bar{\sigma}_2) = 0 \qquad (7)$$

where the constants A, B, and C are:

$$A = t_1/2 + t_2 + t_3$$
$$B = t_2/2 + t_3$$
$$C = t_3/2 \qquad (8)$$

Solution of these equations requires that the determinant.

$$\begin{vmatrix} A & B & C & \sum_1^2 \bar{\sigma}T^2/6 \\ 1 & 1 & 1 & 0 \\ 1/t_1 & -1/t_2 & 0 & -(\bar{\sigma}_1 + \bar{\sigma}_2) \\ 0 & 1/t_2 & -1/t_3 & -(\bar{\sigma}_3 + \bar{\sigma}_2) \end{vmatrix} = 0 \qquad (9)$$

In using the equations $\bar{\sigma}_1$ and $\bar{\sigma}_3$ are calculated using curvature measurements and the following equation

$$\bar{\sigma} = Et/2R \qquad (10)$$

where E is the elastic modulus, t the section thickness and R the radius curvature. Thus $\bar{\sigma}_2$, can then be calculated from equation (9).

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